4/28/17 "Life shrinks or expands in proportion to one's courage." -Anais Nin

HW: "A2 CC Q4 T1 Review"
Test 1 on Tuesday 5/2

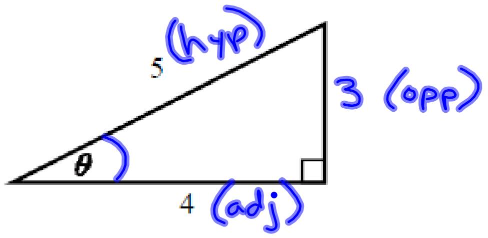
AIM: What are the Sine and Cosine functions?

Warm Up:

SOH CAH TOA

1. For each of the right triangles below, state the values of $\sin(\theta)$, $\cos(\theta)$, and $\tan(\theta)$.

(a)

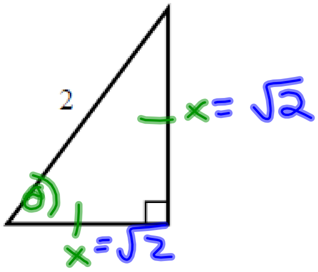


$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

$\tan\theta = \frac{3}{4}$

(b)



$x^2 + x^2 = 2^2$

$2x^2 = 4$

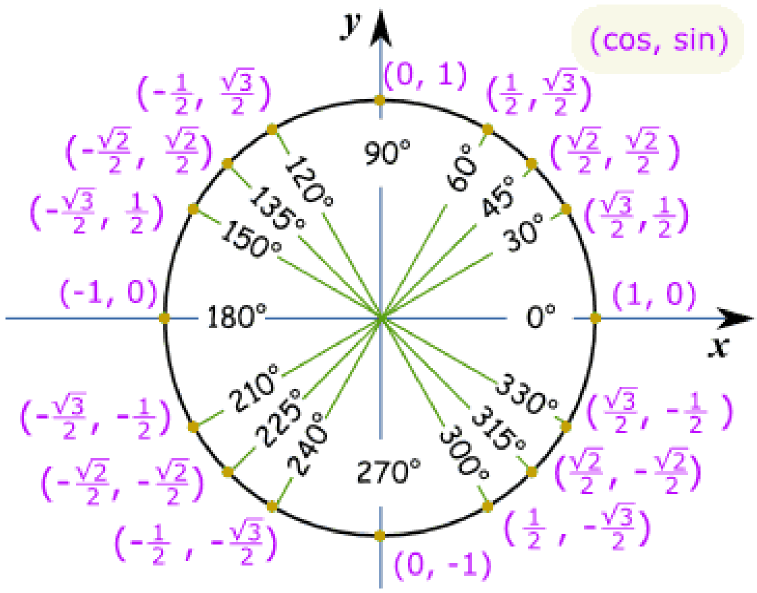
$x^2 = 2$

$x = \sqrt{2}$

$\sin\theta = \frac{\sqrt{2}}{2}$

$\cos\theta = \frac{\sqrt{2}}{2}$

$\tan\theta = \frac{\sqrt{2}}{\sqrt{2}} = 1$



y
x
(cos, sin)
(0, 1)
(1/2, √3/2)
(√2/2, √2/2)
(√3/2, 1/2)
(1, 0)
(√3/2, -1/2)
(√2/2, -√2/2)
(1/2, -√3/2)
(0, -1)
(-1/2, -√3/2)
(-√2/2, -√2/2)
(-√3/2, -1/2)
(-1, 0)
(-√3/2, 1/2)
(-√2/2, √2/2)
(-1/2, √3/2)
0°
30°
45°
60°
90°
120°
135°
150°
180°
210°
225°
240°
270°
300°
315°
330°

The sine and cosine functions form the basis of trigonometry. We would like to define them so that their definition is consistent with what you already are familiar with concerning right triangle trigonometry. Recall from Common Core Geometry that in a right triangle the sine and cosine ratios were defined as:

$$\sin A = \frac{\text{side length opposite of } A}{\text{length of the hypotenuse}} \qquad \cos A = \frac{\text{side length adjacent to } A}{\text{length of the hypotenuse}}$$

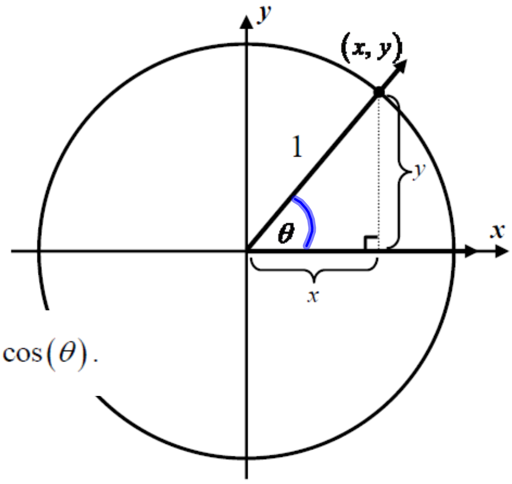
$\boxed{\text{SOH CAH TOA}}$

$$\tan\theta = \frac{\text{side opposite}}{\text{side adjacent}}$$

***Exercise* #1:** Consider the **unit circle** shown below with an angle, θ, drawn in standard position.

(a) Given the right triangle shown, find an expression for $\sin(\theta)$.

$\sin\theta = \frac{y}{1} = y$



(b) Given the right triangle shown, find an expression for $\cos(\theta)$.

$\cos\theta = \frac{x}{1} = x$

(b) Given the right triangle shown, find an expression for $\tan(\theta)$.

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

We thus define the sine and cosine functions by using the coordinates on the unit circle. They are the first functions that are **geometrically defined** as they are based on the geometry of a circle (circular functions).

THE DEFINITION OF THE SINE AND COSINE FUNCTIONS

For an angle in standard position whose terminal ray passes through the point (x, y) on the unit circle:

$\sin(\theta)=$ the y-coordinate and $\cos(\theta)=$ the x-coordinate

(cos, sin)

sin between -1 and 1

cos between -1 and 1

THE DEFINITION OF TANGENT IN TERMS OF SINE AND COSINE

$$\tan\theta=\frac{\sin(\theta)}{\cos(\theta)}$$

The above definition is **unquestionably the most important fact to memorize** concerning trigonometry. We can now use this along with our work on the unit circle to determine certain **exact** values of cosine and sine.

Exercise #2: Using the unit circle diagram, determine each of the following values.

(a) $\sin(30^\circ)=$ y value at 30° $\boxed{\frac{1}{2}}$

(b) $\sin(240^\circ)=$ $-\frac{\sqrt{3}}{2}$

(c) $\cos(90^\circ)=$ $\boxed{0}$

(d) $\cos(180^\circ)=$ $\boxed{-1}$

(e) $\sin(90^\circ)=$ $\boxed{1}$

(f) $\sin(135^\circ)=$ $\frac{\sqrt{2}}{2}$

(g) $\cos(150^\circ)=$ $-\frac{\sqrt{3}}{2}$

(h) $\cos(0^\circ)=$ 1

(i) $\tan(60^\circ)$ $\frac{\sin 60}{\cos 60}$ $\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$ $\frac{\sqrt{3}}{\cancel{2}}\cdot\frac{\cancel{2}}{1}=\sqrt{3}$

(j) $\tan(45^\circ)$ $\frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}=1$

(k) $\tan(150^\circ)$ $\frac{\frac{1}{2}}{\frac{-\sqrt{3}}{2}}$ $\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{-\sqrt{3}}=\frac{-1}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}}$ $\frac{-\sqrt{3}}{3}$

(l) $\tan(180^\circ)$ $\frac{0}{-1}=\boxed{0}$

Exercise #3: The terminal ray of an angle, α, drawn in standard position passes through the point $(-0.6, 0.8)$, which lies on the unit circle. Which of the following gives the value of $\sin(\alpha)$?

(1) 1.2 (3) −0.6

(2) 0.8 (4) 0.2

(cos, sin)

It is important to be able to determine the sign (positive or negative) of each of the two basic trigonometric functions for an angle whose terminal ray lies in a given quadrant. The next exercise illustrates this process.

***Exercise* #4:** For each quadrant below, determine if the sine, cosine and tangent of an angle whose terminal ray falls in the quadrant is positive $(+)$ or negative $(-)$.

	I	II	III	IV
$\cos(\theta)$	+	−	−	+
$\sin(\theta)$	+	+	−	−
$\tan(\theta)$	+	−	+	−

Sin	All
Tan	Cos

Since each point on the unit circle must satisfy the equation $x^2 + y^2 = 1$, we can now state what is known as the **Pythagorean Identity**.

THE PYTHAGOREAN IDENTITY

For any angle, θ, $(\cos\theta)^2 + (\sin\theta)^2 = 1$

***Exercise* #5:** An angle, α, has a terminal ray that falls in the second quadrant. If it is known that $\sin(\alpha) = \frac{3}{5}$, determine the value of $\cos(\alpha)$.

$$(\cos)^2 + \left(\frac{3}{5}\right)^2 = 1$$

$$\cos^2 + \frac{9}{25} = 1$$

$$-\frac{9}{25} \qquad -\frac{9}{25}$$

$$\cos^2 = \frac{16}{25}$$

$$\cos = \pm\sqrt{\frac{16}{25}}$$

$$\cos = \pm\frac{4}{5}$$

$$\cos\alpha = -\frac{4}{5}$$

***Exercise* #6:** An angle, θ, has a terminal ray that falls in the first quadrant and $\cos(\theta) = \frac{1}{3}$. Determine the value of $\sin(\theta)$ in simplest radical form

***Exercise* #7:** Consider the angle $\theta = 90^\circ$ or $\frac{\pi}{2}$ radians.

(a) State the values of sine and cosine at this angle.

(b) Why would the value of $\tan(90^\circ)$ be undefined?

***Exercise* #8:** At which of the following angles is tangent undefined?

(1) $\theta = 0^\circ$

(2) $\theta = 270^\circ$

(3) $\theta = 120^\circ$

(4) $\theta = -180^\circ$

On a final note, it is interesting that if we know sine or cosine of an angle and the quadrant of the angle we can find the other two missing trigonometric values.

***Exercise* #9:** Determine the value of $\cos(\theta)$ and $\tan(\theta)$ if $\sin(\theta) = \frac{5}{13}$ and the terminal ray of θ lies in the second quadrant.

***Exercise* #10:** If $\cos(\theta) = a$, where $a > 0$, and the terminal ray of θ lies in the fourth quadrant, then which of the following gives the value of $\tan(\theta)$ in terms of a.

(1) $\frac{1-a}{a}$

(2) $-\sqrt{1-a^2}$

(3) $\sqrt{1-a^2}$

(4) $\frac{-\sqrt{1-a^2}}{a}$